

Calculation Policy

Guildhall Feoffment Primary School

2019

This policy has been largely adapted from the White Rose Maths Hub Calculation Policy with further material added. It is a working document and will be revised and amended as necessary.

It is recommended that children begin with the concrete for each concept and move through the pictorial to the abstract but the steps are fluid and can be used alongside each other to represent the same concept in different ways.

'From concrete manipulatives and experiences, students are guided to uncover abstract mathematical concepts or results... The role of the teacher is that of a facilitator who guides students through the concrete, pictorial and abstract levels of understanding by providing appropriate scaffolding and feedback.'

Ministry of Education (2012)

“You know you’ve mastered something when you can apply it to a totally new problem in an unfamiliar situation or context.”
(Mastering Mathematics, Dr Helen Drury)

*Mastery is not just an assessment grade –
it is an approach to learning, based on high expectations and access for all.*

Statement of Intent

As a community primary school centred in the middle of a bustling town, we ensure that our mathematics curriculum is accessible for pupils of all abilities, backgrounds and beliefs and will maximise the development of every child’s ability and academic achievement. We strive to deliver a range of exciting and engaging maths lessons which are designed to develop children’s fluency, mathematical reasoning and aptitude to solve increasingly complex problems across a range of contexts. We know that providing a variety of contexts for mathematical working allows children to make rich connections across mathematical ideas and other subjects, identifying opportunities to apply skills in science, PE and other areas of the curriculum. We intend pupils to gain an understanding of how mathematics has been developed over centuries, providing the solution to some of history’s most intriguing problems, right through to its importance to their own futures today: crucial to science, technology and engineering and essential for financial literacy and employment in their adult lives. At Guildhall Feoffment, we reject the notion that some children ‘cannot do maths’ and foster a resilient growth mindset; we establish a safe and nurturing classroom environment where young mathematicians are confident and encouraged to take risks to delve deeper into mathematical understanding. We carefully use a variety of assessment techniques to monitor progress and inform teaching, ensuring children are involved in this process and that they are aware of their progress and targets. We intend to make mathematics part of school life, providing opportunities for working across year groups to solve problems and celebrate each other’s achievements in assemblies. We use the combination of these approaches to deliver an effective and engaging mathematics curriculum that installs an appreciation for the complexity and potential of maths and a deep enjoyment and curiosity about the subject.

GUIDANCE- ALL YEAR GROUPS

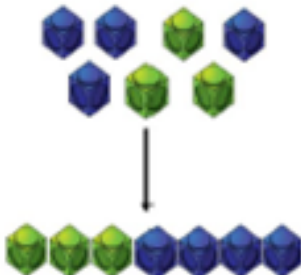
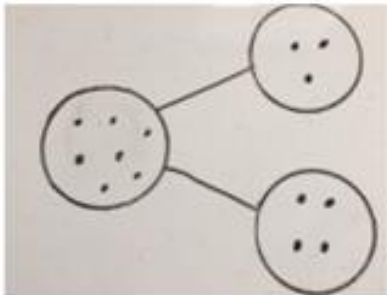
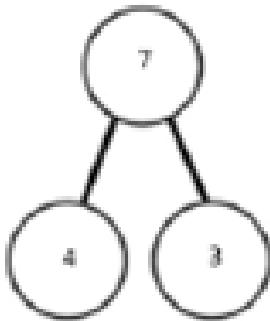
	EYFS/Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	<p>Combining two parts to make a whole: part whole model</p> <p>Starting at the bigger number and counting on- using cubes.</p> <p>Regrouping to make 10 using ten frame.</p>	<p>Adding three single digits.</p> <p>Use of base 10 to combine two numbers.</p>	<p>Column method- regrouping.</p> <p>Using place value counters (up to 3 digits).</p>	<p>Column method- regrouping.</p> <p>(up to 4 digits)</p>	<p>Column method- regrouping.</p> <p>Use of place value counters for adding decimals.</p>	<p>Column method- regrouping.</p> <p>Abstract methods.</p> <p>Place value counters to be used for adding decimal numbers.</p>
Subtraction	<p>Taking away ones</p> <p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10 using the ten frame</p>	<p>Counting back</p> <p>Find the difference</p> <p>Part whole model</p> <p>Make 10</p> <p>Use of base 10</p>	<p>Column method with regrouping.</p> <p>(up to 3 digits using place value counters)</p>	<p>Column method with regrouping.</p> <p>(up to 4 digits)</p>	<p>Column method with regrouping.</p> <p>Abstract for whole numbers.</p> <p>Start with place value counters for decimals- with the same amount of decimal places.</p>	<p>Column method with regrouping.</p> <p>Abstract methods.</p> <p>Place value counters for decimals- with different amounts of decimal places.</p>

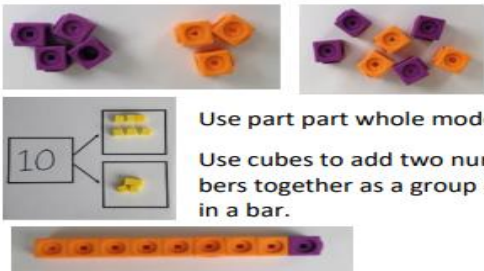
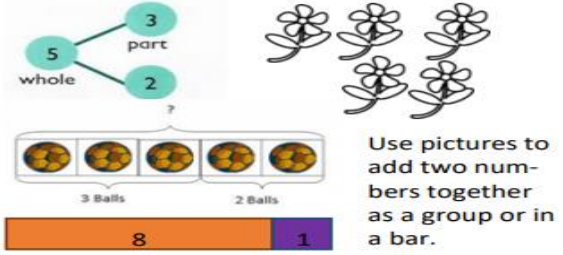
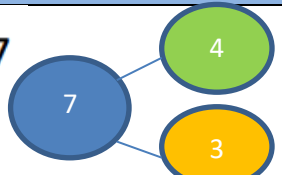

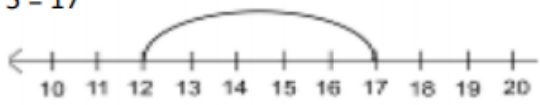
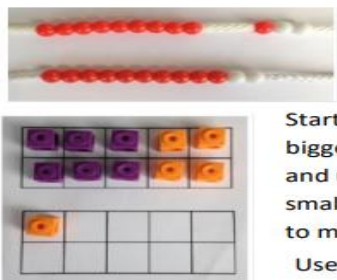
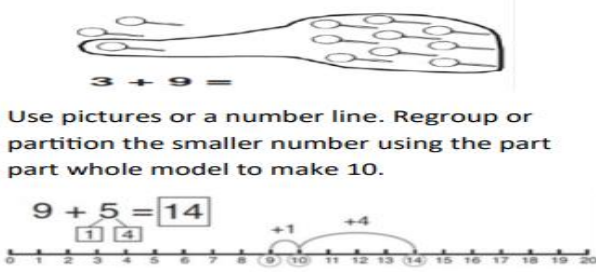

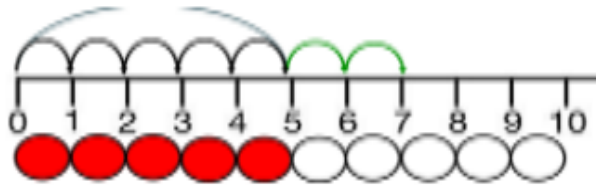
GUIDANCE- ALL YEAR GROUPS


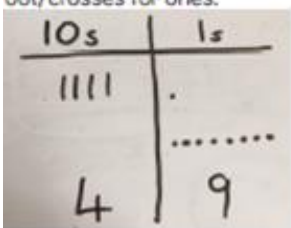
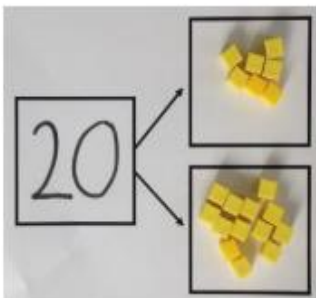
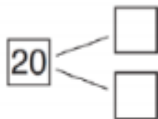

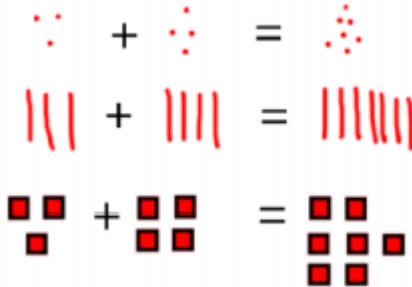
<p>Multiplication</p>	<p>Recognising and making equal groups.</p> <p>Doubling</p> <p>Counting in multiples Use cubes, Numicon and other objects in the classroom</p>	<p>Arrays- showing commutative multiplication</p>	<p>Arrays</p> <p>2d × 1d using base 10</p>	<p>Column multiplication- introduced with place value counters.</p> <p>(2 and 3 digit multiplied by 1 digit)</p>	<p>Column multiplication</p> <p>Abstract only but might need a repeat of year 4 first (up to 4 digit numbers multiplied by 1 or 2 digits)</p>	<p>Column multiplication</p> <p>Abstract methods (multi-digit up to 4 digits by a 2 digit number)</p>
<p>Division</p>	<p>Sharing objects into groups</p> <p>Division as grouping e.g. I have 12 sweets and put them in groups of 3, how many groups?</p> <p>Use cubes and draw round 3 cubes at a time.</p>	<p>Division as grouping</p> <p>Division within arrays- linking to multiplication</p> <p>Repeated subtraction</p>	<p>Division with a remainder- using lollipop sticks, times tables facts and repeated subtraction.</p> <p>2d divided by 1d using base 10 or place value counters</p>	<p>Division with a remainder</p> <p>Short division (up to 3 digits by 1 digit- concrete and pictorial)</p>	<p>Short division</p> <p>(up to 4 digits by a 1 digit number including remainders)</p>	<p>Short division</p> <p>Long division with place value counters (up to 4 digits by a 2 digit number)</p> <p>Children should exchange into the tenths and hundredths column too</p>

POLICY- INCLUDING CONCRETE, PICTORIAL AND ABSRATCT METHODS AND CPONCEPTUAL VARIATIONS



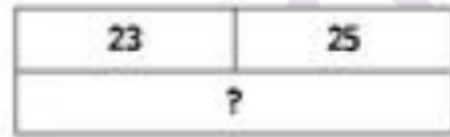
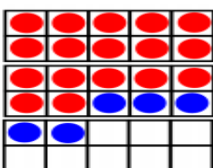
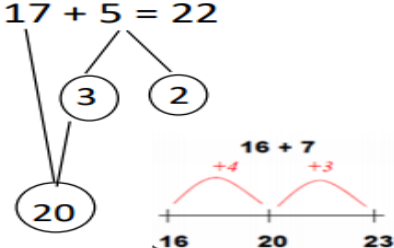
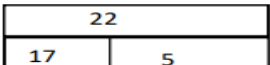

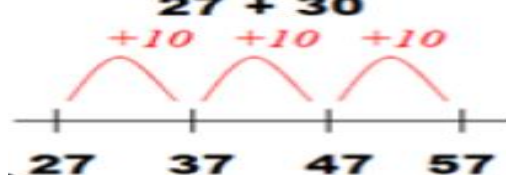

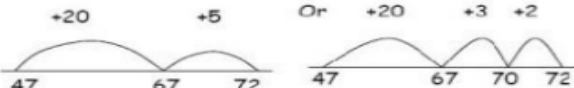
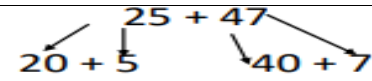

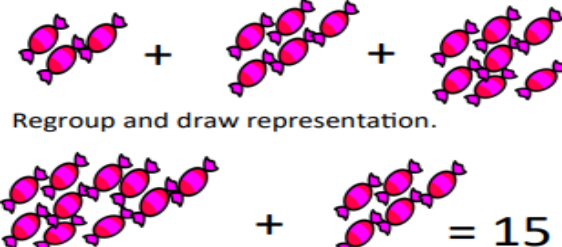
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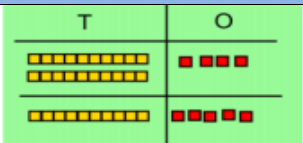
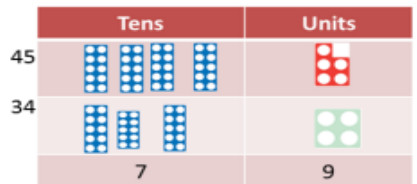
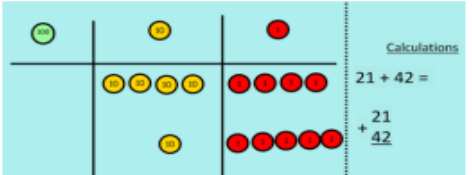
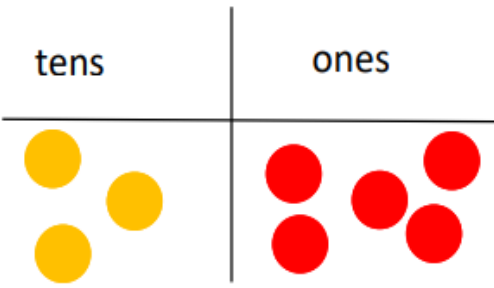
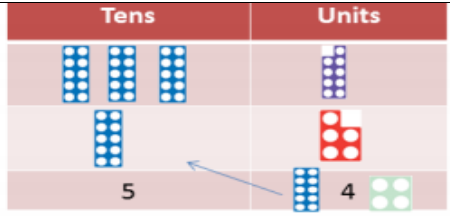
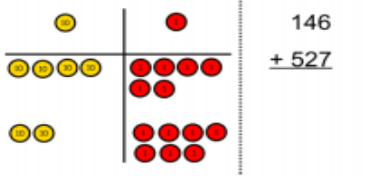
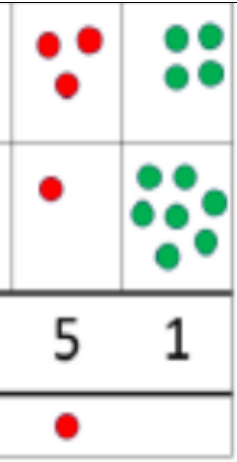
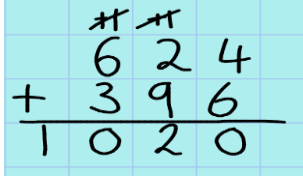
Objective and strategy	Concrete	Pictorial	Abstract
<p>Combining two parts to make a whole: part whole model.</p> <p>Starting at the bigger number and counting on using cubes.</p>	<p>Combining two parts to make a whole – use other resources e.g. shells, teddy bears, cars etc.</p> 	<p>Children to represent cubes/other resources using dots or crosses. Could move to putting each part on a part-whole model.</p> 	<p>$4 + 3 = 7$</p> <p>Four is a part, 3 is a part and seven is the whole.</p> 

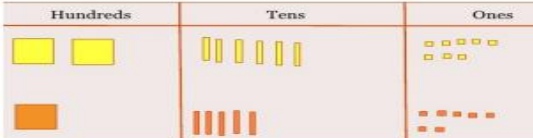
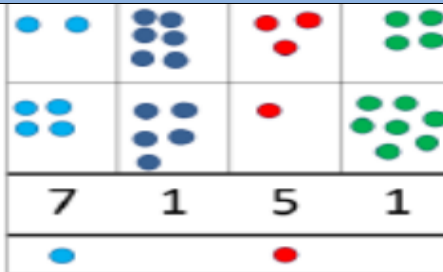
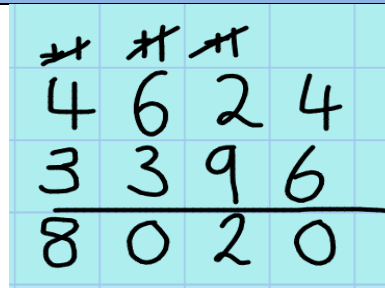
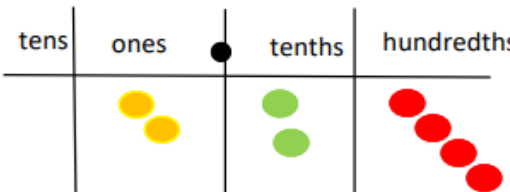
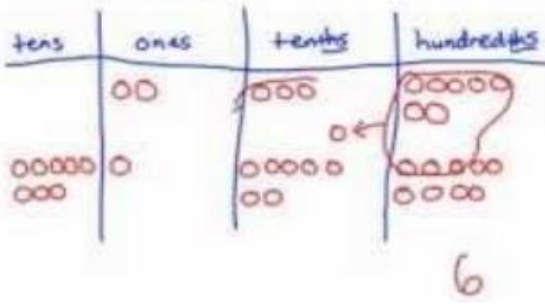
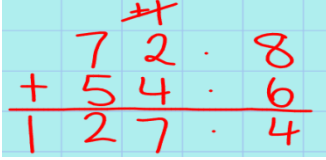
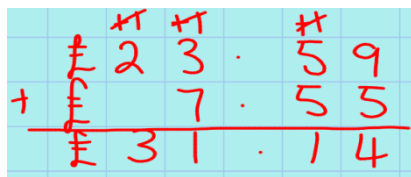
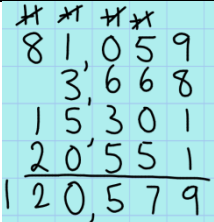

Objective and strategy	Concrete	Pictorial	Abstract
Combining two parts to make a whole: part-whole model.	 <p>Use part part whole model.</p> <p>Use cubes to add two numbers together as a group or in a bar.</p>	 <p>Use pictures to add two numbers together as a group or in a bar.</p>	<p>$4 + 3 = 7$</p>  <p>Use the part-part whole diagram as shown above to move into the abstract.</p> <p>$10 = 6 + 4$</p>
Start at bigger number and counting on	 <p>Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.</p>	<p>$12 + 5 = 17$</p>  <p>Start at the larger number on the number line and count on in ones or in one jump to find the answer.</p>	<p>$5 + 12 = 17$</p> <p>Place the larger number in your head and count on the smaller number to find your answer.</p>
Regrouping to make 10 <i>Essential for column addition later</i>	 <p>$6 + 5 = 11$</p> <p>Start with the bigger number and use the smaller number to make 10.</p> <p>Use ten frames.</p>	 <p>Use pictures or a number line. Regroup or partition the smaller number using the part part whole model to make 10.</p> <p>$9 + 5 = 14$</p>	<p>Children to develop an understanding of equality e.g.</p> <p>$6 + \square = 11$</p> <p>$6 + 5 = 5 + \square$</p> <p>$6 + 5 = \square + 4$</p>
Represent & use number bonds for related subtraction	 <p>2 more than 5.</p>		<p>Emphasis should be on the language</p> <p>'1 more than 5 is equal to 6.'</p> <p>'2 more than 5 is 7.'</p> <p>'8 is 3 more than 5.'</p>

Objective and strategy	Concrete	Pictorial	Abstract
Adding multiples of 10	<p>50 = 30 + 20</p>  <p>Model using dienes and bead strings</p>	<p>Children to represent the base 10 e.g. lines for tens and dot/crosses for ones.</p> 	$20 + 30 = 50$ $70 = 50 + 20$ $40 + \square = 60$
Use known number facts: part part whole	 <p>Children explore ways of making numbers within 20</p>	 $\square + \square = 20$ $20 - \square = \square$ $\square + \square = 20$ $20 - \square = \square$	$\square + 1 = 16$ $16 - 1 = \square$ $1 + \square = 16$ $16 - \square = 1$
Using known facts		 <p>Children draw representations of H, T and O</p>	$3 + 4 = 7$ <i>leads to</i> $30 + 40 = 70$ <i>leads to</i> $300 + 400 = 700$

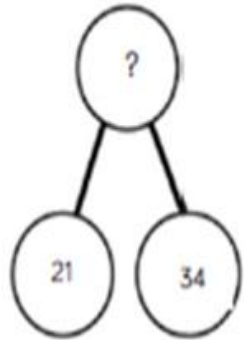
ADDITION +

Bar modelling	 $3 + 4 = 7$	 $7 + 3 = 10$	 $23 + 25 = 48$
Add a two digit number and ones	 <p>17 + 5 = 22 Use ten frame to make 'magic ten'</p> <p>Children explore the pattern. 17 + 5 = 22 27 + 5 = 32</p>	<p>Use part part whole and number line to model.</p> <p>17 + 5 = 22</p> 	<p>17 + 5 = 22</p> <p>Explore related facts</p> <p>17 + 5 = 22 5 + 17 = 22 22 - 17 = 5 22 - 5 = 17</p> 
Add a 2 digit number and tens	 $25 + 10 = 35$ <p>Explore that the ones digit does not change</p>	<p>27 + 30</p> 	$27 + 10 = 37$ $27 + 20 = 47$ $27 + \square = 57$
Add two 2 digit numbers	 <p>Model using dienes , place value counters and numicon</p>	 <p>Use number line and bridge ten using part whole if necessary.</p>	<p>25 + 47</p>  $20 + 40 = 60$ $5 + 7 = 12$ $60 + 12 = 72$
Add three 1 digit numbers	 <p>Combine to make 10 first if possible, or bridge 10 then add third digit</p>	 <p>Regroup and draw representation.</p> $4 + 7 + 6 = 15$	<p>4 + 7 + 6 = 10 + 7</p> <p>10</p> <p>= 17</p> <p>Combine the two numbers that make/ bridge ten then add on the third.</p>

Objective and strategy	Concrete	Pictorial	Abstract
Column addition- no regrouping (friendly numbers)	 <p>Model using Dienes or Numicon</p> <p>Add together the ones first, then the tens.</p>  <p>Move to using place value counters</p>  <p>Calculations</p> $\begin{array}{r} 21 \\ + 42 \\ \hline 63 \end{array}$	<p>Children move to drawing the counters using a tens and one frame.</p> 	$\begin{array}{r} 223 \\ + 114 \\ \hline 337 \end{array}$ <p>Add the ones first, then the tens, then the hundreds.</p>
Column addition- with regrouping	 <p>Exchange ten ones for a ten. Model using Numicon and pv counters.</p>  <p>Calculations</p> $\begin{array}{r} 39 \\ + 15 \\ \hline 54 \end{array}$	 <p>Children can draw a representation of the grid to further support their understanding, carrying the ten <u>underneath</u> the line</p>	<p>Children can be supported by portioning numbers first so they can understand the regrouping.</p> <p>NOTE: All regrouping should be recorded above the top row of numbers with an addition symbol and crossed off when used. This supports children when moving onto long multiplication in UKS2. See below</p> 

Objective and strategy	Concrete	Pictorial	Abstract
<p>Year 4: Add numbers with up to 4 digits</p>	<p>Children continue to use dienes or pv counters to add, exchanging ten ones for a ten, ten tens for a hundred and ten hundreds for a thousand.</p> 	 <p>Draw representations using pv grid.</p>	 <p>Continue from previous work to regroup hundreds. Relate to money and measures.</p>
<p>Year 5: Add numbers with more than 4 digits.</p> <p>Add decimals with 2 decimal places, including money.</p>	<p>As year 4</p>  <p>Introduce decimal place value counters and model exchange for addition.</p>	<p>2.37 + 81.79</p> 	 
<p>Y6: Add several numbers of increasing complexity Including adding money, measure and decimals with different numbers of decimal points.</p>	<p>See Y5</p>	<p>See Y5</p>	  <p>Insert place holders after decimals</p>

Conceptual variation- different ways to ask the children to solve $21 + 34$



?	
21	34

Word problems:

In year 3, there are 21 children and in year 4, there are 34 children.
How many children in total?

$21 + 34 = 55$. Prove it

21

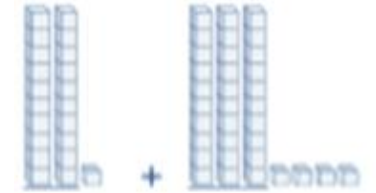
+34

—

$21 + 34 =$

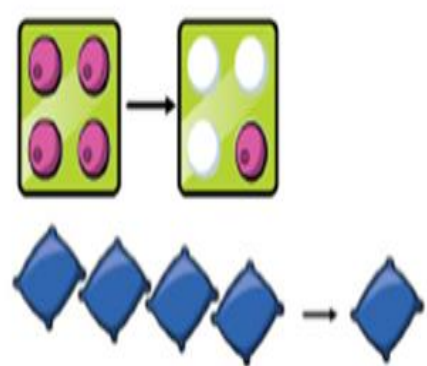
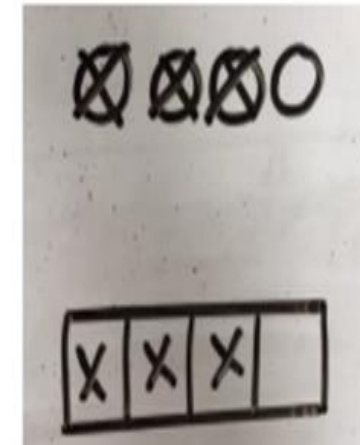
 $= 21 + 34$

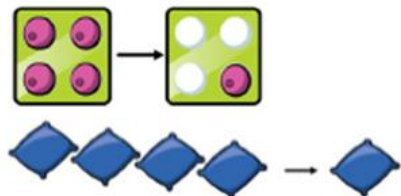
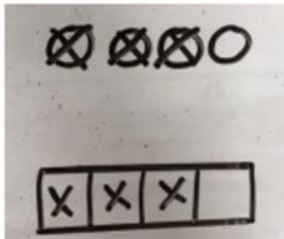
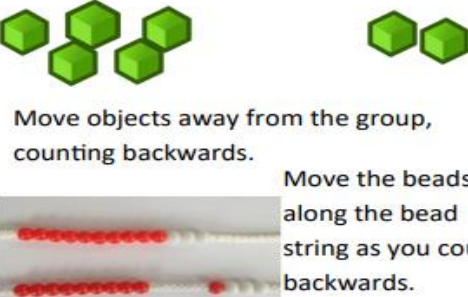
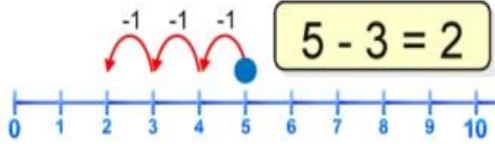
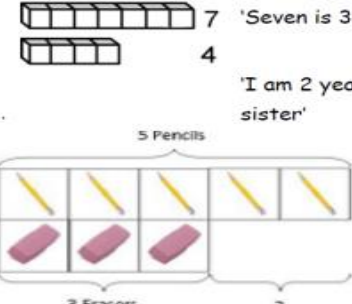
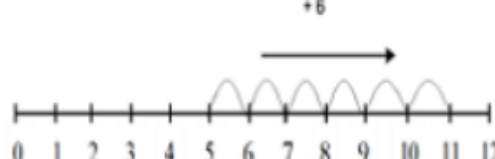
Calculate the sum of twenty-one and thirty-four.

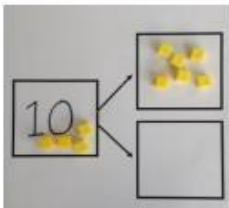
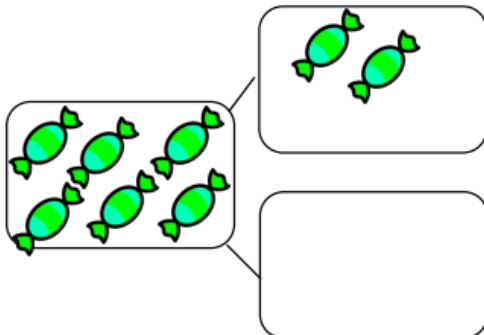
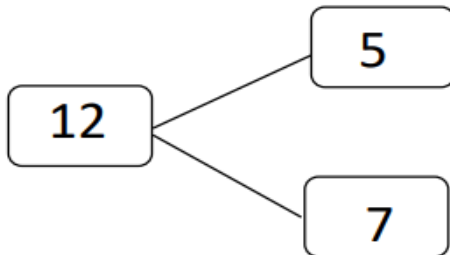
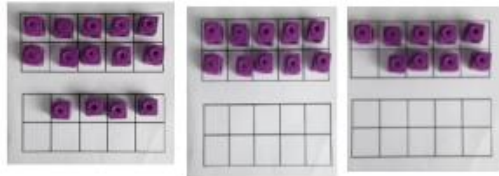
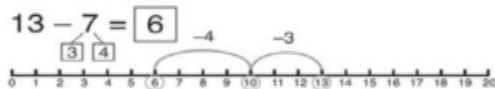




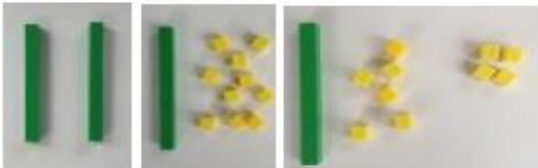



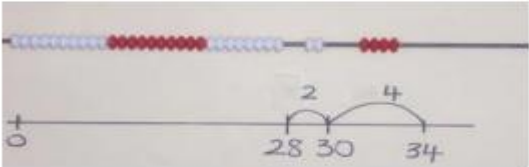
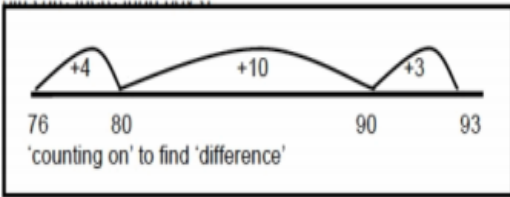
Missing digit problems:

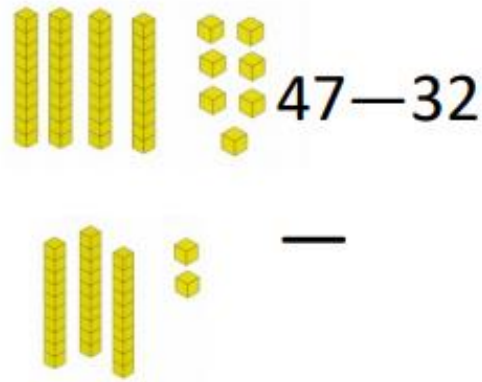
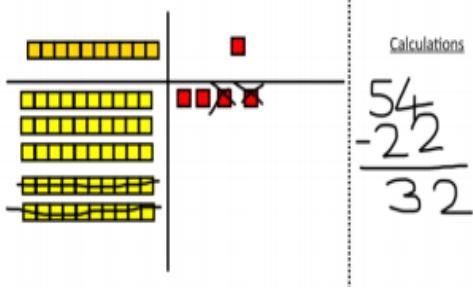

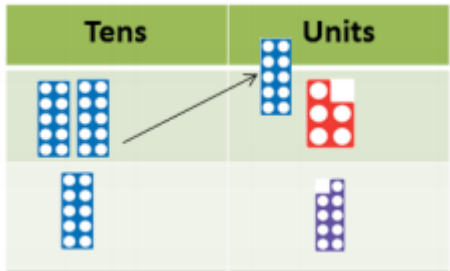
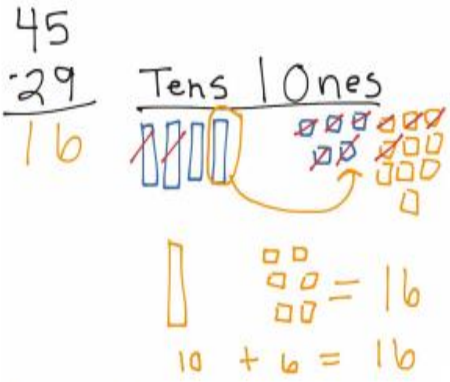
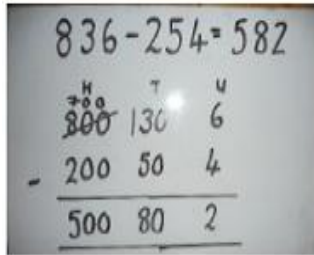
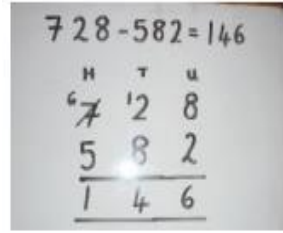
10s	1s
	?
?	5

Objective and strategy	Concrete	Pictorial	Abstract
Taking ones away	<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	

Objective and strategy	Concrete	Pictorial	Abstract
Taking ones away	<p>Physically taking away and removing objects from a whole (ten frames, Numicon, cubes and other items such as beanbags could be used).</p> <p>$4 - 3 = 1$</p> 	<p>Children to draw the concrete resources they are using and cross out the correct amount. The bar model can also be used.</p> 	<p>$7 - 4 = 3$</p> <p>$16 - 9 = 7$</p>
Counting back	 <p>Move objects away from the group, counting backwards.</p> <p>Move the beads along the bead string as you count backwards.</p>	 <p>Count back in ones using a number line.</p>	<p>Put 13 in your head, count back 4. What number are you at?</p>
Find the difference	<p>Compare objects and amounts</p>  <p>Lay objects to represent bar model.</p> <p>'Seven is 3 more than four'</p> <p>'I am 2 years older than my sister'</p>	<p>Count on using a number line to find the difference.</p> 	<p>James has 14 sweets and his brother has 5. How many more does James have than his brother?</p>

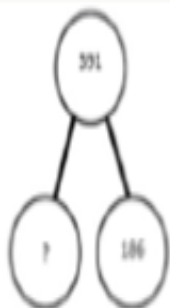
Represent and use number bonds and related subtraction facts within 20 Part Part Whole model	 <p>Link to addition. Use PPW model to model the inverse.</p> <p>If 10 is the whole and 6 is one of the parts, what is the other part?</p> $10 - 6 = 4$	 <p>Use pictorial representations to show the part.</p>	Move to using numbers within the part whole model. 		
Make 10	$14 - 9$  <p>Make 14 on the ten frame. Take 4 away to make ten, then take one more away so that you have taken 5.</p>	$13 - 7$  <p>Jump back 3 first, then another 4. Use ten as the stopping point.</p>	$16 - 8$ <p>How many do we take off first to get to 10? How many left to take off?</p>		
Bar model	 $5 - 2 = 3$	 $10 - 2 = 8$	<table border="1" data-bbox="1606 1007 2110 1088"><tr><td>8</td><td>2</td></tr></table> $10 = 8 + 2$ $10 = 2 + 8$ $10 - 2 = 8$ $10 - 8 = 2$	8	2
8	2				

Objective and strategy	Concrete	Pictorial	Abstract
Regroup a ten into ten ones	 <p>Use a PV chart to show how to change a ten into ten ones, use the term 'take and make'</p>	 <p>$20 - 4 =$</p>	$20 - 4 = 16$
Partitioning to subtract without regrouping. 'Friendly numbers'	<p>$34 - 13 = 21$</p>  <p>Use Dienes to show how to partition the number when subtracting without regrouping.</p>	<p>Children draw representations of Dienes and cross off.</p>  <p>$43 - 21 = 22$</p>	$43 - 21 = 22$
Make 10 strategy <i>Progression should be crossing one ten, crossing more than one ten, crossing the hundreds.</i>	 <p>$34 - 28$</p> <p>Use a bead bar or bead strings to model counting to next ten and the rest.</p>	 <p>Use a number line to count on to next ten and then the rest.</p>	$93 - 76 = 17$

Objective and strategy	Concrete	Pictorial	Abstract
Column subtraction without regrouping (friendly numbers)	 <p>47—32</p> <p>Use base 10 or Numicon to model</p>	 <p>Calculations</p> $\begin{array}{r} 54 \\ - 22 \\ \hline 32 \end{array}$ <p>Draw representations to support understanding</p>	$47 - 24 = 23$ $\begin{array}{r} 40 + 7 \\ - 20 + 4 \\ \hline 20 + 3 \end{array}$ <p>Intermediate step may be needed to lead to clear subtraction understanding.</p> 
Column subtraction with regrouping	 <p>Begin with base 10 or Numicon. Move to pv counters, modelling the exchange of a ten into ten ones. Use the phrase 'take and make' for exchange.</p>	 <p>Children may draw base ten or PV counters and cross off.</p>	 <p>Begin by partitioning into pv columns</p>  <p>Then move to formal method.</p>

Objective and strategy	Concrete	Pictorial	Abstract
Subtracting tens and ones. Year 4 subtract with up to 4 digits. <i>Introduce decimal subtraction through context of money</i>	<p>234 - 179</p> <p>Model process of exchange using Numicon, base ten and then move to PV counters.</p>	Children to draw pv counters and show their exchange—see Y3	<p>Can use the phrase 'take and make' for exchange</p>
Year 5- Subtract with at least 4 digits, including money and measures. <i>Subtract with decimal values, including mixtures of integers and decimals and aligning the decimal</i>	As Year 4	Children to draw pv counters and show their exchange—see Y3	<p>Use zeros for place-holders.</p>
Year 6—Subtract with increasingly large and more complex numbers and decimal values.	Children struggling with concepts use Y4 and 5 guidance to support (differentiation)	Children struggling with concepts use Y4 and 5 guidance to support (differentiation)	

Conceptual variation- different ways to ask the children to solve 391- 186



391	
186	?

Raj spent £391, Timmy spent £186.
How much more did Raj spend?

Calculate the difference between 391 and 186.

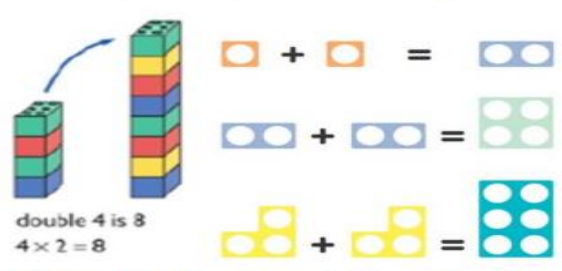

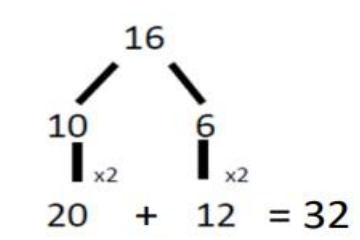
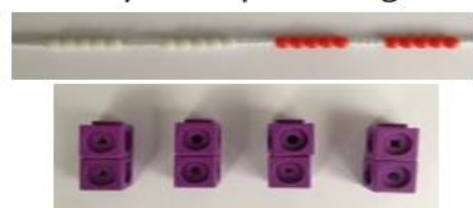
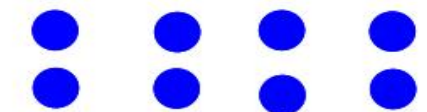



$$\square = 391 - 186$$

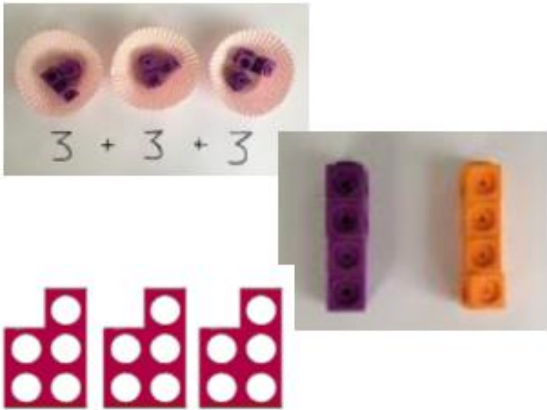
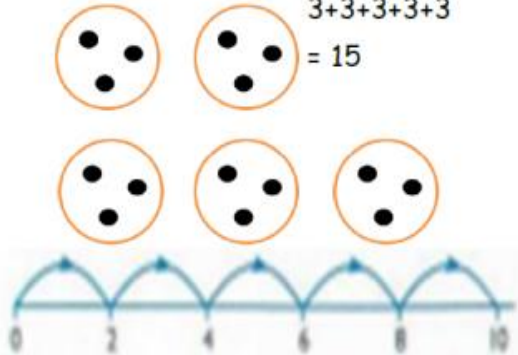

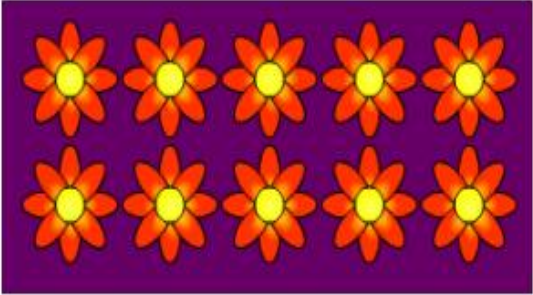
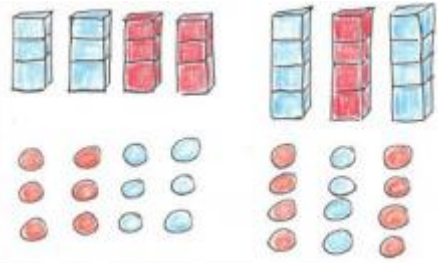
$$\begin{array}{r} 391 \\ -186 \\ \hline \end{array}$$

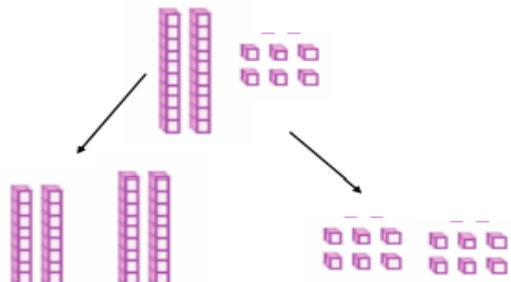

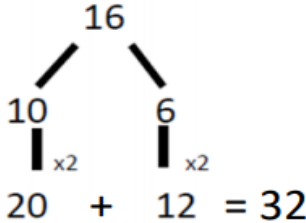
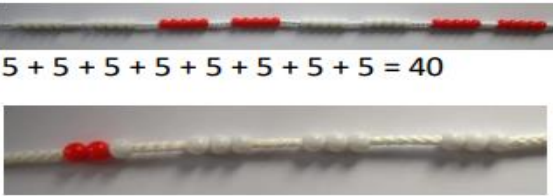
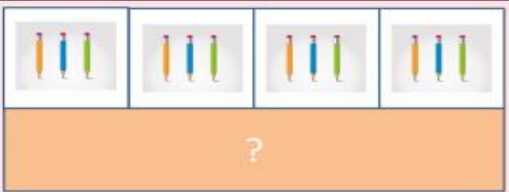
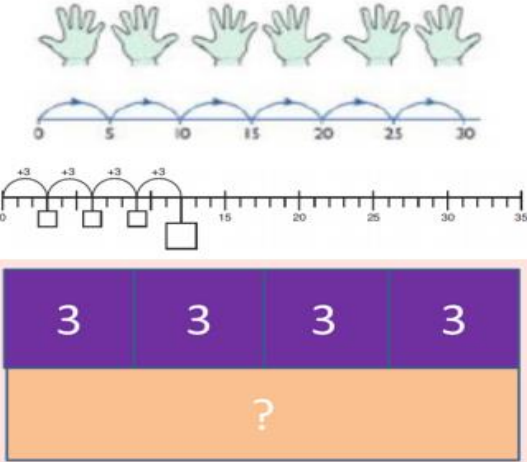
What is 186 less than 391?



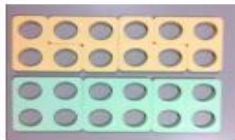


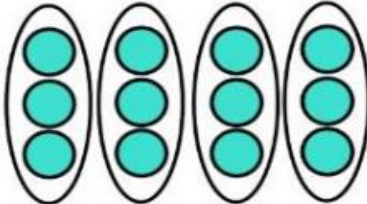
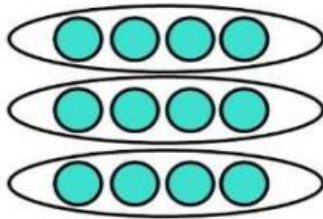


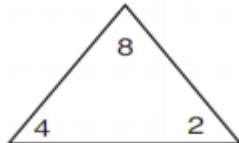
Missing digit calculations

$$\begin{array}{r} 39\square \\ -\square\square6 \\ \hline \square05 \end{array}$$

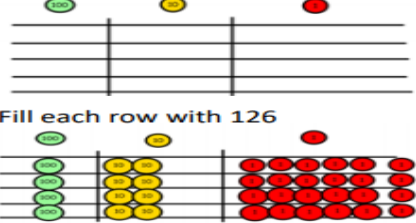
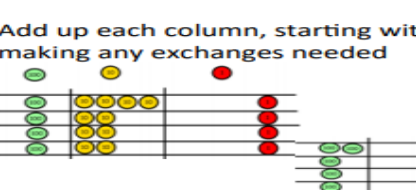
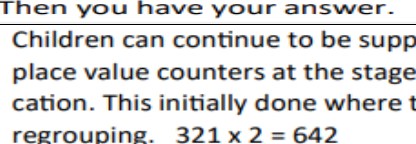
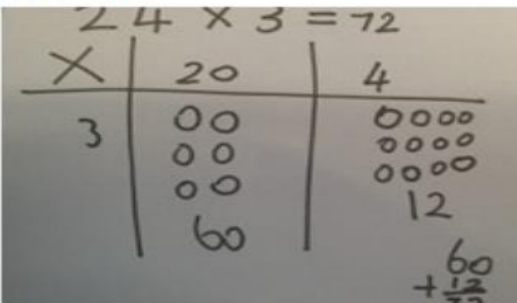
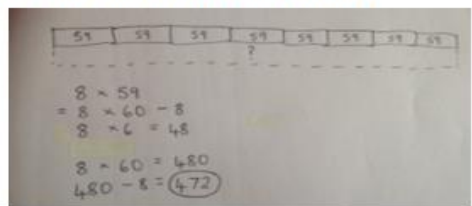
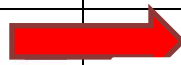

Objective and strategy	Concrete	Pictorial	Abstract
Doubling	<p>Use practical activities using manipulatives including cubes and Numicon to demonstrate doubling</p>  <p>double 4 is 8 $4 \times 2 = 8$</p>	<p>Draw pictures to show how to double numbers</p> <p>Double 4 is 8</p> 	<p>Partition a number and then double each part before recombining it back together.</p>  <p>$20 + 12 = 32$</p>
Counting in multiples	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting.</p> 	 <p>Children make representations to show counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>2, 4, 6, 8, 10</p> <p>5, 10, 15, 20, 25, 30</p>
Making equal groups and counting the total	  <p>$\square \times \square = 8$</p> <p>Use manipulatives to create equal groups.</p>	<p>Draw pictures to represent amounts and make groups</p>	<p>$2 \times 4 = 8$</p>

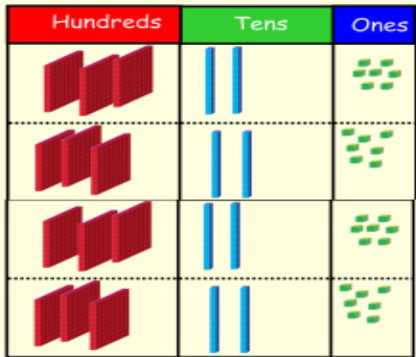
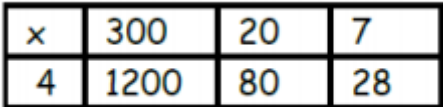
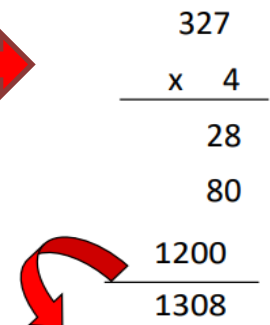
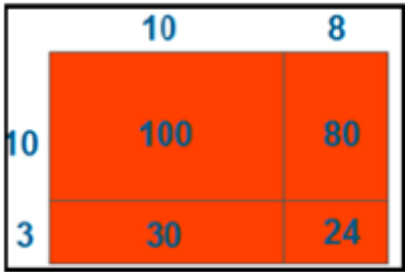
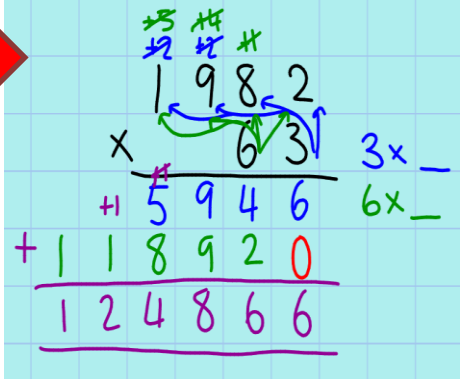
Objective and strategy	Concrete	Pictorial	Abstract
Repeated addition	 <p>Use different objects to add equal groups</p>	<p>Use pictorial including number lines to solve prob There are 3 sweets in one bag. How many sweets are in 5 bags altogether?</p> <p>$3+3+3+3+3 = 15$</p> 	<p>Write addition sentences to describe objects and pictures.</p>  <p>$2+2+2+2+2 = 10$</p>
Understanding arrays	<p>Use objects laid out in arrays to find the answers to 2 lots 5, 3 lots of 2 etc.</p> 	 <p>Draw representations of arrays to show understanding</p>	<p>$3 \times 2 = 6$</p> <p>$2 \times 5 = 10$</p>

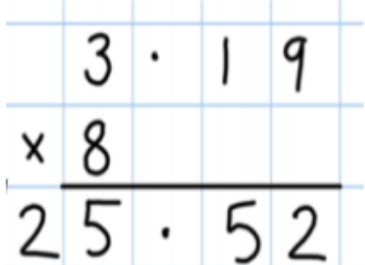
Objective and strategy	Concrete	Pictorial	Abstract
Doubling	<p>Model doubling using dienes and PV counters.</p>  $40 + 12 = 52$	<p>Draw pictures and representations to show how to double numbers</p>  <p><i>Double 6 is 12</i></p>	<p>Partition a number and then double each part before recombining it back together.</p>  $20 + 12 = 32$
Counting in multiples of 2, 3, 4, 5, 10 from 0 (repeated addition)	<p>Count the groups as children are skip counting, children may use their fingers as they are skip counting. Use bar models.</p>  $5 + 5 + 5 + 5 + 5 + 5 + 5 + 5 = 40$ 	<p>Number lines, counting sticks and bar models should be used to show representation of counting in multiples.</p> 	<p>Count in multiples of a number aloud.</p> <p>Write sequences with multiples of numbers.</p> <p>0, 2, 4, 6, 8, 10</p> <p>0, 3, 6, 9, 12, 15</p> <p>0, 5, 10, 15, 20, 25, 30</p> $4 \times 3 = \square$

Objective and strategy	Concrete	Pictorial	Abstract																				
Multiplication is commutative	<p>Create arrays using counters and cubes and Numicon.</p>    <p>Pupils should understand that an array can represent different equations and that, as multiplication is commutative, the order of the multiplication does not affect the answer.</p>  	<p>Use representations of arrays to show different calculations and explore commutativity.</p>  	<p>$12 = 3 \times 4$</p> <p>$12 = 4 \times 3$</p> <p>Use an array to write multiplication sentences and reinforce repeated addition.</p>  <p>$5 + 5 + 5 = 15$ $3 + 3 + 3 + 3 + 3 = 15$ $5 \times 3 = 15$ $3 \times 5 = 15$</p>																				
Using the Inverse	<p><i>This should be taught alongside division, so pupils learn how they work alongside each other.</i></p> 	 <table><tr><td><input type="text"/></td><td>\times</td><td><input type="text"/></td><td>$=$</td><td><input type="text"/></td></tr><tr><td><input type="text"/></td><td>\times</td><td><input type="text"/></td><td>$=$</td><td><input type="text"/></td></tr><tr><td><input type="text"/></td><td>\div</td><td><input type="text"/></td><td>$=$</td><td><input type="text"/></td></tr><tr><td><input type="text"/></td><td>\div</td><td><input type="text"/></td><td>$=$</td><td><input type="text"/></td></tr></table>	<input type="text"/>	\times	<input type="text"/>	$=$	<input type="text"/>	<input type="text"/>	\times	<input type="text"/>	$=$	<input type="text"/>	<input type="text"/>	\div	<input type="text"/>	$=$	<input type="text"/>	<input type="text"/>	\div	<input type="text"/>	$=$	<input type="text"/>	<p>$2 \times 4 = 8$ $4 \times 2 = 8$ $8 \div 2 = 4$ $8 \div 4 = 2$ $8 = 2 \times 4$ $8 = 4 \times 2$ $2 = 8 \div 4$ $4 = 8 \div 2$</p> <p>Show all 8 related fact family sentences.</p>
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Objective and strategy	Concrete	Pictorial	Abstract
Grid method	<p>Show links with arrays to introduce grid method.</p> <div><div><div><div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></div><div></d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Objective and strategy	Concrete	Pictorial	Abstract																													
<p>Grid method recap from year 3 for 2 digits x 1 digit</p> <p>Move to multiplying 3 digit numbers by 1 digit. (year 4 expectation)</p>	<p>Move on to place value counters to show how we are finding groups of a number. We are multiplying by 4 so we need 4 rows</p>  <p>Fill each row with 126</p>  <p>Add up each column, starting with the ones making any exchanges needed</p>  <p>Then you have your answer.</p>	<p>Children can represent their work with place value counters in a way that they understand.</p> <p>They can draw the counters using colours to show different amounts or just use the circles in the different columns to show their thinking as shown below.</p> 	<p>Start with multiplying by one digit numbers and showing the clear addition alongside the grid.</p> <table border="1" data-bbox="1733 437 2069 564"><tr><td>x</td><td>30</td><td>5</td></tr><tr><td>7</td><td>210</td><td>35</td></tr></table> <p>210 + 35 = 245</p>	x	30	5	7	210	35																							
x	30	5																														
7	210	35																														
Column multiplication	<p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. 321 x 2 = 642</p> <table border="1" data-bbox="434 963 826 1331"><tr><th>Hundreds</th><th>Tens</th><th>Ones</th></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>3</td><td>2</td><td>1</td></tr><tr><td>3</td><td>2</td><td>1</td></tr></table> <p>It is important at this stage that they always multiply the ones first.</p> <p>The corresponding long multiplication is modelled alongside</p>	Hundreds	Tens	Ones	3	2	1	3	2	1	3	2	1	3	2	1	<table border="1" data-bbox="1155 844 1527 920"><tr><td>x</td><td>300</td><td>20</td><td>7</td></tr><tr><td>4</td><td>1200</td><td>80</td><td>28</td></tr></table> <p>The grid method may be used to show how this relates to a formal written method.</p>  <p>Bar modelling and number lines can support learners when solving problems with multiplication alongside the formal written methods.</p>	x	300	20	7	4	1200	80	28	 <table data-bbox="1800 836 1957 1059"><tr><td>327</td></tr><tr><td>x 4</td></tr><tr><td>1308</td></tr></table>  <table border="1" data-bbox="1688 1233 1890 1377"><tr><td>327</td></tr><tr><td>x 4</td></tr><tr><td>1308</td></tr></table> <p>This will lead to a compact method.</p>	327	x 4	1308	327	x 4	1308
Hundreds	Tens	Ones																														
3	2	1																														
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4	1200	80	28																													
327																																
x 4																																
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1308																																

Objective and strategy	Concrete	Pictorial	Abstract
<p>Column Multiplication for 3 and 4 digits x 1 digit.</p>	 <p>It is important at this stage that they always multiply the ones first.</p> <p>Children can continue to be supported by place value counters at the stage of multiplication. This initially done where there is no regrouping. $321 \times 2 = 642$</p>		 <p>This will lead to a compact method.</p>
<p>Column multiplication</p>	<p>Manipulatives may still be used with the corresponding long multiplication modelled alongside.</p>	 <p>Continue to use bar modelling to support problem solving</p>	 <p>Place zero (red) first so it is not forgotten. Ensures this rows answer is multiplied by ten as they are multiplying by 60 not 6. Short multiplication for each row. Add two results for final answer.</p>

Objective and strategy	Concrete	Pictorial	Abstract
Multiplying decimals up to 2 decimal places by a single digit.	Manipulatives such as pv counters can be used to support working	Pictorial methods can be used to support working	Remind children that the single digit belongs in the units column. Line up the decimal points in the question and the answer. 

Conceptual variation – ways of asking children to solve 6×23

23	23	23	23	23	23
<div></div>					
?					

Mai had to swim 23 lengths, 6 times a week.
How many lengths did she swim in one week?

With the counters, prove that $6 \times 23 = 138$



Find the product of 6 and 23

$6 \times 23 =$

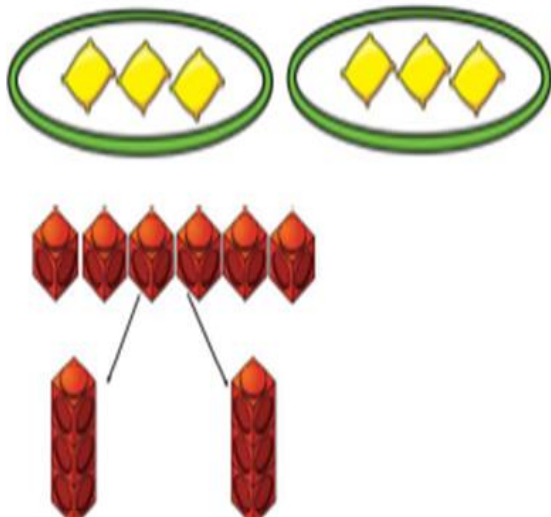
$= 6 \times 23$

6	23
$\times 23$	$\times 6$
—	—


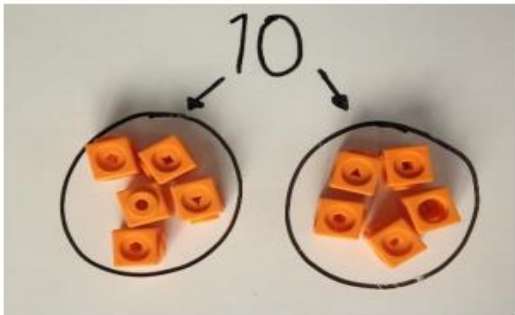
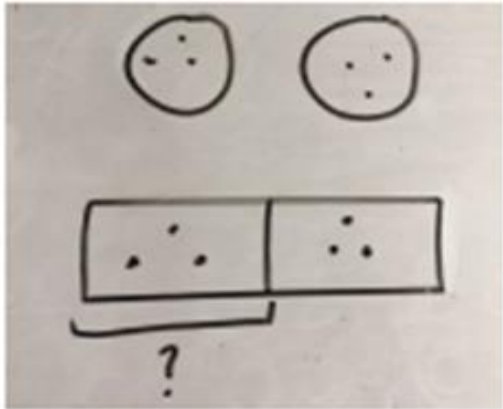
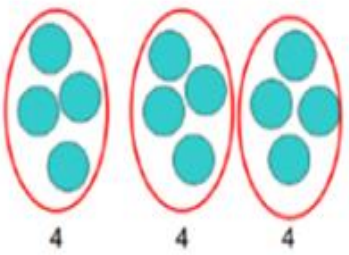
What is the calculation?
What is the product?

100s	10s	1s
		

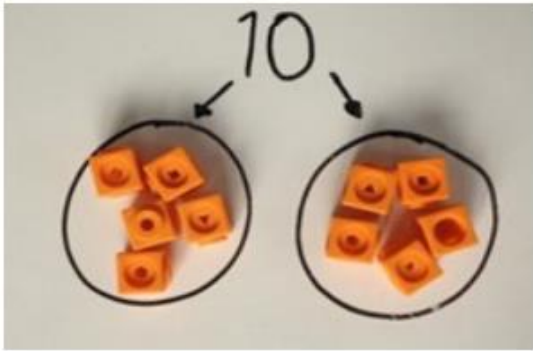

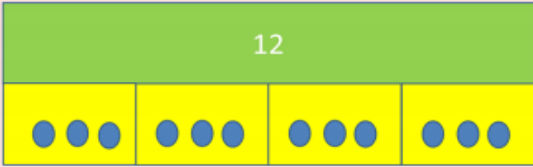


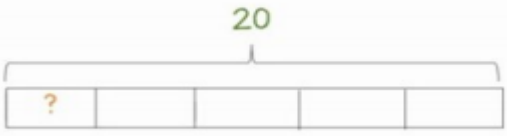
DIVISION ÷





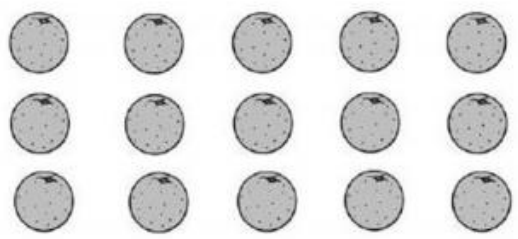
Objective and strategy	Concrete	Pictorial	Abstract
Sharing using a range of objects	$6 \div 2 =$ 		

DIVISION ÷

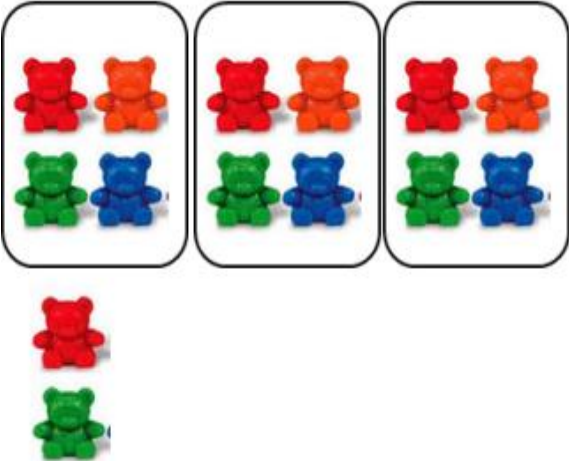
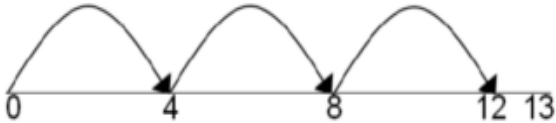

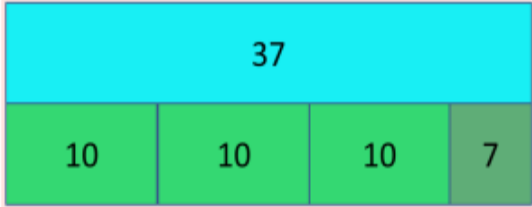
Objective and strategy	Concrete	Pictorial	Abstract
Division as sharing	  <p>I have 10 cubes, can you share them equally in 2 groups?</p>	 <p>Sharing:</p>  <p>12 shared between 3 is 4</p>	<p>12 shared between 3 is 4</p>

DIVISION ÷


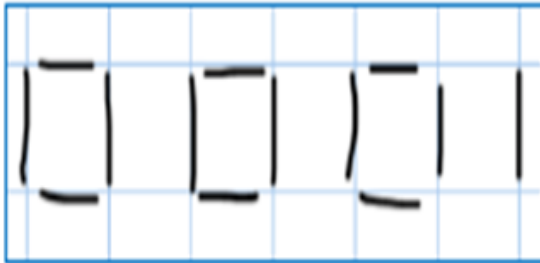
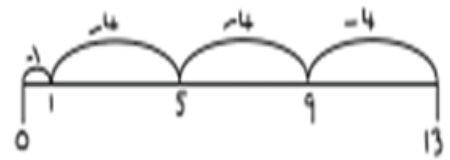
Objective and strategy	Concrete	Pictorial	Abstract
Division as sharing	 <p>I have 10 cubes, can you share them equally in 2 groups?</p>	<p>Children use pictures or shapes to share quantities</p>  $8 \div 2 = 4$ <p>Children use bar modelling to show and support understanding</p>  $12 \div 4 = 3$	$12 \div 3 = 4$
Division as grouping	<p>Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.</p>  	<p>Use number lines and bar model for grouping</p> <p>Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.</p>  $20 \div 5 = ?$ $5 \times ? = 20$	$28 \div 7 = 4$ <p>Divide 28 into 7 groups. How many are in each group?</p>

Objective and strategy	Concrete	Pictorial	Abstract
Division as grouping	<p>Use cubes, counters, objects or place value counters to aid understanding.</p>  <p>24 divided into groups of 6 = 4</p> <p>$96 \div 3 = 32$</p> 	<p>Continue to use bar modelling to aid solving division problems.</p>  <p>$20 \div 5 = ?$ $5 \times ? = 20$</p>	<p>How many groups of 6 in 24?</p> <p>$24 \div 6 = 4$</p>
Division with arrays	<p>Link division to multiplication by creating an array and thinking about the number sentences that can be created.</p> <p>Eg $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$</p> 	<p>Draw an array and use lines to split the array into groups to make multiplication and division sentences</p> 	<p>Find the inverse of multiplication and division sentences by creating eight linking number sentences.</p> <p>$7 \times 4 = 28$ $4 \times 7 = 28$ $28 \div 7 = 4$ $28 \div 4 = 7$ $28 = 7 \times 4$ $28 = 4 \times 7$ $4 = 28 \div 7$ $7 = 28 \div 4$</p>


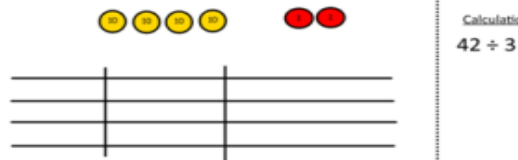

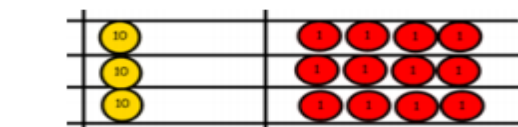
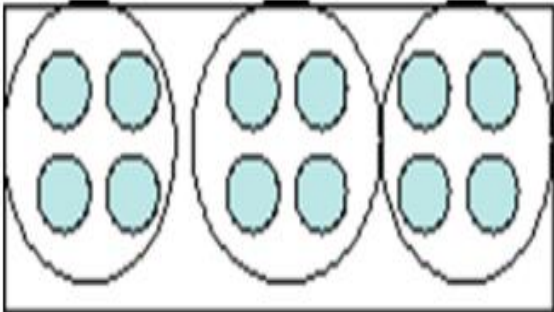
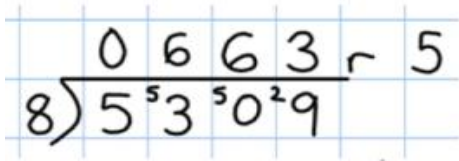
DIVISION ÷

Objective and strategy	Concrete	Pictorial	Abstract
Division with remainders	<p>$14 \div 3 =$ Divide objects between groups and see how much is left over</p> 	<p>Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder.</p>  <p>Draw dots and group them to divide an amount and clearly show a remainder.</p>  <p>Use bar models to show division with remainders.</p> 	<p>Complete written divisions and show the remainder using 'r'.</p> <p>$29 \div 8 = 3 \text{ REMAINDER } 5$</p> <p>↑ ↑ ↑ ↑ dividend divisor quotient remainder</p>


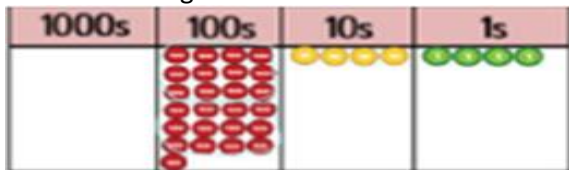
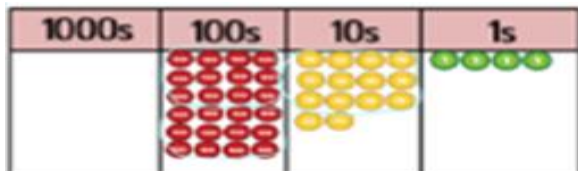
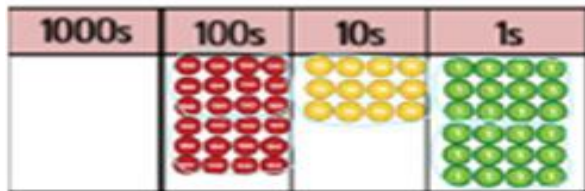
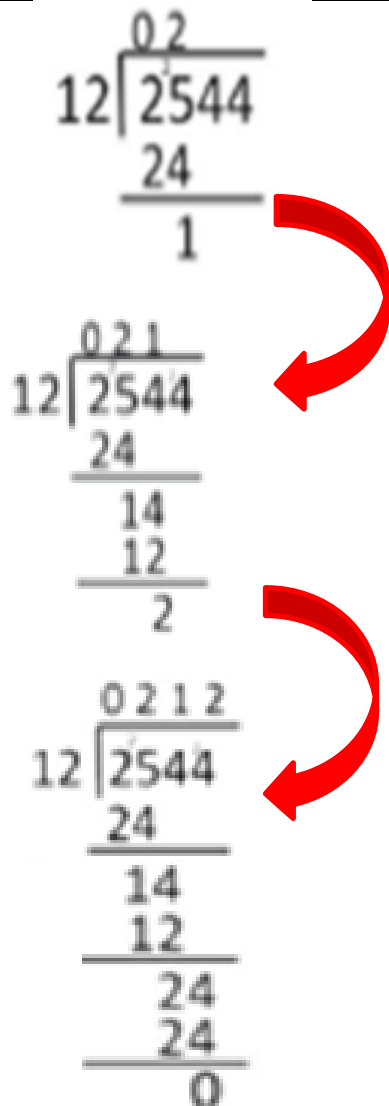
DIVISION ÷

Objective and strategy	Concrete	Pictorial	Abstract
<p>Division with remainders</p>	<p>$2d \div 1d$ with remainders using lollipop sticks (Cuisenaire rods above a ruler can also be used).</p> <p>$13 \div 4 =$</p>  <p>Use of lollipop stick to form wholes-squares are made because we are dividing by 4.</p> <p>There are 3 whole squares with 1 left over. Answer 3 r 1</p>	 <p>Children represent the lollipop sticks pictorially.</p> <p>There are 3 whole squares with 1 left over. Answer 3 r 1</p>	<p>Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.</p>  <p>'3 groups of 4 with 1 left over'</p>

DIVISION ÷

Objective and strategy	Concrete	Pictorial	Abstract
<p>Divide at least 3 digit numbers by 1 digit.</p> <p>Short Division</p>	<p>$96 \div 3$</p> <p>Tens Units</p> <p>3 2</p>  <p>Use place value counters to divide using the bus stop method alongside</p>  <p>42 ÷ 3 =</p> <p>Start with the biggest place value, we are sharing 40 into three groups. We can put 1 ten in each group and we have 1 ten left over</p>  <p>We exchange this ten for ten ones and then share the ones equally among the groups.</p>  <p>We look how much in 1 group so the answer is 14.</p>	<p>Students can continue to use drawn diagrams with dots or circles to help them divide numbers into equal groups.</p>  <p>Encourage them to move towards counting in multiples to divide more efficiently.</p>	<p>Begin with divisions that divide equally with no remainder</p> $\begin{array}{r} 218 \\ 3 \overline{) 654} \end{array}$ <p>Move onto divisions with a remainder.</p> $\begin{array}{r} 86 \text{ r } 2 \\ 3 \overline{) 258} \end{array}$ <p>Finally move into decimal places to divide the total accurately.</p> $\begin{array}{r} 14.6 \\ 35 \overline{) 511.0} \end{array}$ 

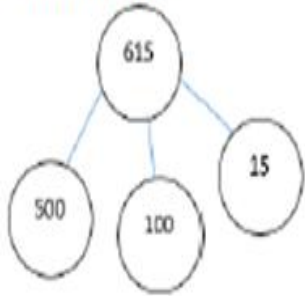
DIVISION ÷

Objective and strategy	Concrete	Pictorial	Abstract
<p>Long division</p> <p>$2544 \div 12 =$</p> <p><i>Children are encouraged to write 'Multiples to Help' down the side of their abstract working.</i></p> <p><i>For calculations with remainders, children work through the method until the number left is smaller than the divisor- this is their remainder.</i></p>	<p>Use place value counters to support</p>  <p>We can't group 2 thousands into groups of 12 so we will exchange them.</p>  <p>We can group 24 hundreds into groups of 12 which leaves us with 1 hundred.</p>  <p>After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.</p>  <p>After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 groups of 12, which leaves no remainder.</p>	<p>Children can draw the place value frames and counters to support their working.</p>	

DIVISION ÷

Conceptual variation – ways of asking children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?



I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$\square = 615 \div 5$$

What is the calculation?
What is the answer?

100s	10s	1s